Distance Transform

Etienne Folio

Technical Report n^o0806, JUNE 2008
revision 1748

Abstract: A distance transform, also known as distance map or distance field, is a representation of a distance function to an object, as an image. Such maps are used in several applications, especially in document image analysis. Some optimizations can be obtained by less generic methods: for example, maps calculated by front propagation can determine shorter paths, assuming that the image is non-convex. This presentation discusses different distance transform algorithms and underlines their advantages and weaknesses. Finally we will explain our choices.

Résumé : Une carte de distances est une représentation sous forme d'image d'une fonction distance à un objet. Ces cartes sont utilisées dans de nombreuses applications, en particulier en analyse d'images de documents qui nous serviront d'illustration. Certaines methodes de calcul de cartes moins generiques que d'autres peuvent s'averer plus rapides : par exemple, des cartes calculées par propagation de fronts permettent de determiner des plus courts chemins mais ne fonctionnent que lorsque le support est connu pour etre non-convexe. Cette présentation fait un tour d'horizon des différents algorithmes de calculs de cartes de distance, met en évidence leurs atouts et faiblesses et explique les choix retenus.

Keywords
MILENA, C++, Image processing, Distance transform, Distance map, Closest point, Influence zones

Laboratoire de Recherche et Développement de l'Epita
14-16, rue Voltaire – F-94276 Le Kremlin-Bicêtre cedex – France
Tél. +33 1 53 14 59 47 – Fax. +33 1 53 14 59 22
folio@lrde.epita.fr – http://www.lrde.epita.fr/

Copying this document

Contents

Introduction

A propos

This paper is divided in three main parts. It will begin by a little reminder of some vocabulary about image processing and a quick explaination of the utility of the distance transformation algorithms. In a second part, we will see how a distance map can be computed with different approaches: the naive one, and two more sophisticated algorithms. In the last part, we will present our enhancements and use of these algorithms.

Acknowledgments

I thank Dr. Thierry Geraud and Roland Levillain for all the guidance throughout the project.

I also thank the entire laboratoy for their support and help.

Chapter 1

Preliminaries

1.1 Vocabulary

1.1.1 Convex set

In Euclidean space, an object is convex if for every pair of points within the object, every point on the straight line segment that joins them is also within the object.

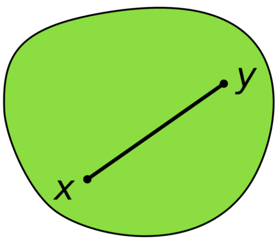

Figure 1.1: A convex set.

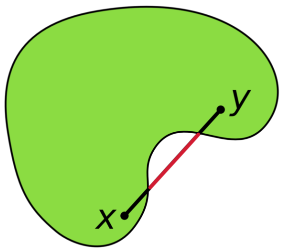

Figure 1.2: A non-convex set.

1.1.2 Connected space

A connected space is a topological space which cannot be represented as the disjoint union of two or more nonempty open subsets.

We often call a connected space an “object”.

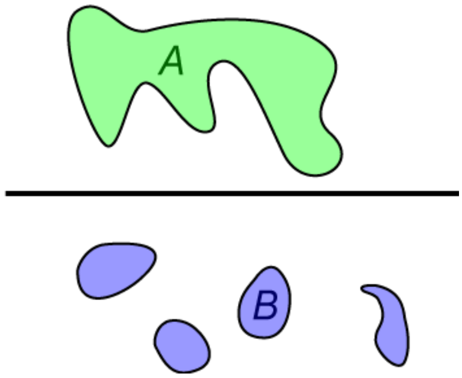

Figure 1.3: Connected and disconnected subspaces of $\mathbb{R}^2$.

1.1.3 Euclidean distance

The Euclidean distance or Euclidean metric is the "ordinary" distance between two points that one would measure with a ruler, which can be proven by repeated application of the Pythagorean theorem.

For 2D points, $P = (p_x, p_y)$ and $Q = (q_x, q_y)$, the Euclidean distance is computed as:

$$\sqrt{(p_x - q_x)^2 + (p_y - q_y)^2}$$

In Euclidean n-space, it is defined as:

$$\sqrt{\sum_{i=1}^{n} (p_i - q_i)^2}$$

1.1.4 Geodesic distance

In the mathematical field of graph theory, the distance between two vertices in a graph is the number of edges in a shortest path connecting them. This is also known as the geodesic distance.

In a image using (for example) Manhattan distance, each point is assimilated as a node of a graph, and each linkage between them and their neighbors is assimilated as an arc. By this way, it is possible to determine a geodesic distance in a connected space of this image.

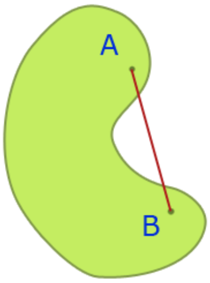

Figure 1.4: Euclidean distance.

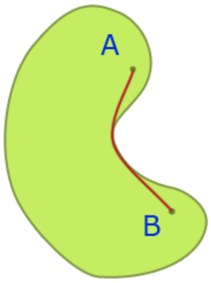

Figure 1.5: Geodesic distance.

1.1.5 Isotropy and anisotropy

Isotropy is uniformity in all directions. On the opposite, the anisotropy is the property of being directionally dependent.

1.1.6 Manhattan distance

The manhattan distance is a metric in which the distance between two points is the sum of the (absolute) differences of their coordinates. It is also known as rectilinear distance, L_1 distance or city block distance.

For example, in the plane, the manhattan distance between the point P_1 with coordinates (x_1, y_1) and the point P_2 at (x_2, y_2) is:

$$D_{Manhattan} = |x_1 - x_2| + |y_1 - y_2|$$

.

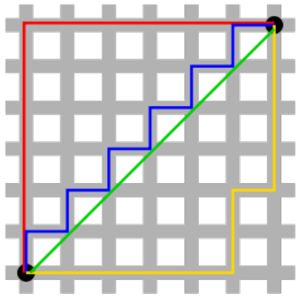

Figure 1.6: The red, blue, and yellow lines have the same length (12) using both Euclidean and Manhattan distance. Using Euclidean geometry, the green line has length $6 \times \sqrt{2} \approx 8.48$, and is the unique shortest path.

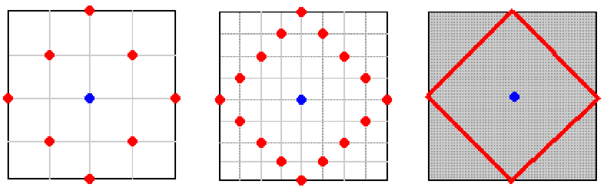

Figure 1.7: Circles using discrete and continuous manhattan distance.

1.1.7 Chessboard distance

The chessboard distance is a metric defined on a vector space where the distance between two vectors is the greatest of their differences along any coordinate dimension.

In two dimensions, i.e. plane geometry, if the points P and Q have cartesian coordinates (x_1, y_1) and (x_2, y_2), their chessboard distance is:

$$D_{Chess} = max(|x_2 - x_1|, |y_2 - y_1|)$$

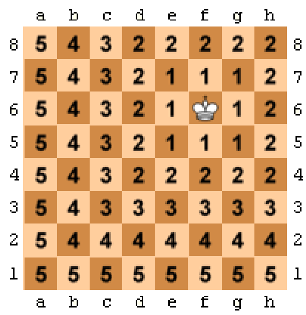

Figure 1.8: The chessboard distance is the number of moves a king requires to move between spaces.

In n-dimensions, the chessboard distance between two vectors or points P and Q, with standard coordinates p_i and q_i, respectively, is:

$$D_{Chess} = max_i(|p_i - q_i|)$$

1.1.8 Distance map

A distance transform, also known as distance map or distance field, is a representation of a distance function to an object, as an image. This means that the map supplies each pixel of the image with the distance to the nearest obstacle pixel.

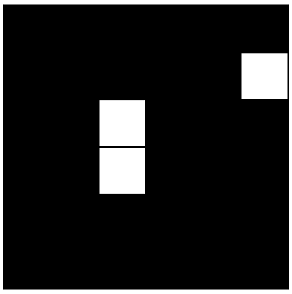

Figure 1.9: Binary input image. Black is background, white is object.

4	3	2	3	2	1
3	2	1	2	1	0
2	1	0	1	2	1
2	1	0	1	2	2
3	2	1	2	3	3
4	3	2	3	4	4

Figure 1.10: Output distance map using Manhattan distance.

Figure 1.11: Distance map using stretched grayscale values for display. The center of the image is the background.

1.1.9 Closest point map

The closest point map associates for each background point the nearest object's point.

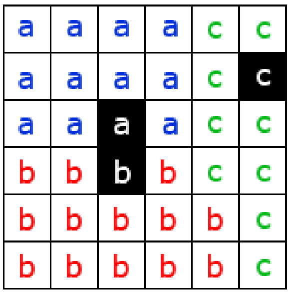

Figure 1.12: Closest point map. Here, we labelized each point in order to see associate points.

1.1.10 Influence zone

The influence zone labelizes each background point of the map to the label of the closest object.

1.2 Utility of the distance map

Chapter 2

Distance map

2.1 Naive approach

2.1.1 The algorithm

2.1.2 Performance

2.2 Chamfer algorithm

2.2.1 The algorithm

2.2.2 Which chamfer to use?

2.2.3 Performance

2.3 Propagation using buckets

2.3.1 The algorithm

2.3.2 Performance

Chapter 3

Our proposals

3.1 Improvements

3.2 Performance

3.3 Selection

3.4 Application

Chapter 4

Related Work

Conclusion

Bibliography