VAUCANSON User's Manual

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Introduction

VAUCANSON is a free software platform dedicated to the manipulation of finite state automata. Here, 'finite state automata' is to be understood in the broadest sense: VAUCANSON supports *weighted* automata over a free monoid, and even *weighted* automata on some *non free monoids* (currently only products of free monoids—a.k.a. transducers—are supported).

The platform consists in a couple of components:

- **The Vaucanson library** is a C++ library that implements objects for automata, rational expressions, as well as algorithms on these objects. This library is generic, in the sense that it makes it possible to write an algorithm once and apply it to different types of automata. However this genericity is achieved in a way that should not cause any slowdown at runtime: because the type of the automata manipulated is known at compile time, compiling an algorithm will generate code that is almost as efficient as an algorithm dedicated to this type of automaton.
- **TAF-Kit** is a command-line interface to the library, that allows user to execute VAUCANSON's algorithms without any knowledge of C++. Because the VAUCANSON library needs to know the type of automata at compile time, the TAF-KIT interface has been instantiated for a predefined set of common automaton types.

TAF-KIT does not allow to write new algorithms nor to manipulate new types of automata, but it makes it possible to combine without efforts a large set of algorithms on common automata types.

A repository of automata that shows examples of automata of various types, and also contains tools to create families of automata.

Ideally this manual should document all of these components. Presently it only documents TAF-KIT because that is the more accessible part of VAUCANSON.

Chapter 1

Administrativia

1.1 Getting Vaucanson

The latest version of the VAUCANSON platform can be downloaded from http://vaucanson.lrde.epita.fr/.

1.2 Licensing

Vaucanson is a free software released under the GNU General Public Licence version 2. If you are unfamiliar with this license, please read the file 'COPYING' (at the root of the source tree) for details.

1.3 Building Vaucanson

Detailed information is provided in the files 'INSTALL', which is generic to all packages using the GNU Build System, and 'doc/README.pdf' which details VAUCANSON's specific build process. The typical installation commands will install VAUCANSON in '/usr/local/'.

```
$ cd vaucanson-1.2.95a
$ ./configure
$ make
$ sudo make install
```

Although we discourage it, you may also use VAUCANSON without installing it: you would have to use -I /full-path-to/vaucanson-1.2.95a/include when compiling C++ programs, and move to directory 'vaucanson-1.2.95a/taf-kit/tests/' to execute TAF-KIT. (This directory contains wrapper around the real TAF-KIT programs from 'taf-kit/tests/' that enable them to run locally.)

1.4 Getting in Touch

Please send any question or comment to vaucanson@lrde.epita.fr, and report bugs to either our issue tracker at http://vaucanson.lrde.org/ or by mail to vaucanson-bugs@lrde.epita.fr.

You can subscribe to these mailing lists at https://www.lrde.epita.fr/cgi-bin/mailman/ listinfo/vaucanson and https://www.lrde.epita.fr/cgi-bin/mailman/listinfo/vaucanson-bugs if you like, but this is not a requirement for sending mails.

Chapter 2

TAF-Kit

TAF-KIT is a command-line interface to VAUCANSON. It is a set of programs that should be called from the shell and that can be used to chain operations on automata.

2.1 TAF-Kit Instances

Because of the generic programming paradigm used in the VAUCANSON library, the types of the automata manipulated have to be known at compile time. TAF-KIT has therefore been compiled for several predefined types of automata. It is actually the same program that is *instantiated* for different kinds of alphabets and weights. Here are the names of these instances, and the kind of automata they represent:

program name	automaton type	alphabet type	weight semiring
vcsn-char-b	automata	characters	$\langle \mathbb{B}, \lor, \land angle$
vcsn-int-b	automata	integers	$\langle \mathbb{B}, \lor, \land angle$
vcsn-char-z	automata	characters	$\langle \mathbb{Z}, +, imes angle$
vcsn-char-zmax	automata	characters	$\langle \mathbb{Z}, \max, + \rangle$
vcsn-char-zmin	automata	characters	$\langle \mathbb{Z}, \min, + \rangle$
vcsn-char-r	automata	characters	$\langle \mathbb{R}, +, imes angle$
vcsn-char-char-b	automata	pairs of characters	$\langle \mathbb{B}, \lor, \land angle$
vcsn-char-int-b	automata	pairs of character and integer	$\langle \mathbb{B}, \lor, \land angle$
vcsn-int-int-b	automata	pairs of integers	$\langle \mathbb{B}, \lor, \land angle$
vcsn-char-fmp-b	transducers	characters	$\langle \mathbb{B}, \lor, \land angle$
vcsn-char-fmp-z	transducers	integers	$\langle \mathbb{Z}, +, imes angle$
vcsn-int-fmp-b	transducers	characters	$\langle \mathbb{B}, \lor, \land angle$
vcsn-int-fmp-z	transducers	integers	$\langle \mathbb{Z}, +, \times \rangle$

Many users of automata consider only automata whose transitions are labeled by letters taken in an alphabet, which we call, roughly speaking, *classical* automata or *Boolean* automata. vcsn-char-b is the TAF-KIT instance they should use. A variant of this program, called vcsn-int-b, handles Boolean automata whose letters are integers. Other variants such as vcsn-char-char-b, vcsn-char-int-b, or vcsn-int-int-b, support alphabets of pairs. All these are called Boolean automata because each word is associated to a Boolean *weight*: either the word as accepted and its weight is *true*, or it is not and its weight is *false*.

VAUCANSON actually supports automata with multiplicities, where words can be associated to weights taken in any semiring. For instance vcsn-char-z associates each word to an integer. The previous table show other semirings that can be used as well.

VAUCANSON also supports weighted transducers. These transducers are actually automata over a product two free monoids. In VAUCANSON we call these FMP, for *free monoid products*. The above table lists a few TAF-KIT instance for FMP.

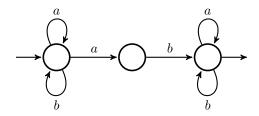


Figure 2.1: The automaton \mathcal{A}_1 , defined over the alphabet $\Sigma = \{a, b\}$. It recognizes any word of Σ^* that contains ab.

2.2 A First Contact

If you have fully installed VAUCANSON (see section 1.3) you should be able to just type any of the following commands and observe their results. If you only compiled VAUCANSON without installing it, you should cd into the 'vaucanson-1.2.95a/taf-kit/tests/' directory and type ./vcsn-char-b instead of vcsn-char-b for each of the following commands.

We are about to play with automaton \mathcal{A}_1 pictured on Figure 2.1. TAF-KIT comes with a set of predefined automata, and \mathcal{A}_1 happens to be one of those: it is called 'a1.xml'. This is a Boolean automaton whose alphabet consists in two characters $\{a, b\}$ so will shall use the vcsn-char-b instance of TAF-KIT.

The following command will just make sure that TAF-KIT knows about this automaton. It will display the number of states, transitions, initial states, and final states of A_1 .

```
$ vcsn-char-b info a1.xml
States: 3
Transitions: 6
Initial states: 1
Final states: 1
```

If you have the GraphViz package installed (see 'doc/README.pdf' for links) you can also display that automaton with:

```
$ vcsn-char-b display a1.xml
```

The displayed automaton won't have a layout as pretty as Figure 2.1, but it represents the same automaton nonetheless.

 \mathcal{A}_1 is a non-deterministic automaton. We could determinize it with the determinize command of TAF-KIT. As most commands of TAF-KIT, determinize produce its output (an XML file representing the automaton) on the standard output, so we will want to divert it to a file.

```
$ vcsn-char-b determinize a1.xml > a1det.xml
$ vcsn-char-b info a1det.xml
States: 4
Transitions: 8
Initial states: 1
Final states: 2
```

The determinized automaton has 4 states and 8 transitions.

Please note that 'aldet.xml' is a file that we just created into the current directory while 'al.xml' is a file that is predefined in VAUCANSON's predefined automata repository. We can call command info on either files using the same syntax because TAF-KIT will look for automata in both places. The command 'vcsn-char-b list-automata' will list all predefined automata for this instance of TAF-KIT.

In the pure Unix tradition, we can of course chain commands with pipes. For instance the above two commands could be rewritten

```
$ vcsn-char-b determinize a1.xml | vcsn-char-b info -
States: 4
Transitions: 8
Initial states: 1
Final states: 2
```

where '-' stands for 'read from standard input'.

TAF-KIT actually supports a more efficient way of chaining commands, called the *internal pipe*. It's called *internal pipe* because the pipe logic is taken care of by TAF-KIT himself, but actually it is not using a Unix pipe at all: the commands are simply serialized in the same process, using the automata object created by the previous one. It is more efficient because the automaton does not have to be converted into XML for output, and then parsed as input of the next command in the chain. Here is how the *internal pipe* looks like; notice how the '|' symbol is protected from its evaluation by the shell.

```
$ vcsn-char-b determinize a1.xml \/ info -
States: 4
Transitions: 8
Initial states: 1
Final states: 2
```

In the above command, '-' does not designate the standard input, it denotes the result of the previous command.

2.3 TAF-Kit's Modus Operandi

All TAF-KIT instances work identically. They just differ on the type of automata they handle, and may offer different algorithms because not all algorithms work on any automata type.

Any time TAF-KIT is run, it breaks its command line into command names and arguments.

The *internal pipe*, $\langle \backslash |'$, is used to separate commands. A command start with a name, is can be followed by several arguments (although only one is used in the above two commands). These arguments can be very different depending on the command. The far we have used filenames as well as '-' (to designate either the standard input or the result of the previous command). Some commands will also accept plain text representing for instance a word or a rational expression.

All commands will also accept some options. There are options to define what the alphabet is, options to define the types to use for input and output, even options to fine-tune how some symbols will be printed. We shall get back to these options later.

For each command, TAF-KIT will

- 1. parse the options
- 2. parse all expected arguments (using indications that may have been given as options)
- 3. execute the algorithm
- 4. print the result (in a format that can be controlled using options)

When commands are chained internally using $\label{eq:commands}$ and $\label{eq:commands}$ are of course omitted.

2.4 Writing Rational Expressions

2.4.1 Rational operators

Any word can be used as a rational expression. Additionally the following operators can be used to combine rational expressions.

e*	Kleen star
e_1e_2	implicit concatenation
$e_1.e_2$	explicit concatenation
e_1 + e_2	disjunction
(<i>e</i>)	grouping

On the alphabet $\Sigma = \{a, b\}$, the language denoted by the rational expression '(a+b)*ab(a+b)*' contains all words that contain 'ab'.

The VAUCANSON library always needs to know on which alphabet a rational expression is defined in order to parse it. This alphabet can be indicated using option '--alphabet=ab' or the shorter form '-aab' (see subsection 2.5.1 for more details).

For instance, here is how to create an automaton that recognizes the same language as (a+b)*ab(a+b)*, and make sure this automaton is equivalent to the automaton \mathcal{A}_1 of Figure 2.1.

```
$ vcsn-char-b exp-to-aut -aab "(a+b)*ab(a+b)*" > aut.xml
$ vcsn-char-b are-equivalent -v aut.xml a1.xml
```

The '-v' is used to request a plain English output from are-equivalent. Without it, TAF-KIT would just set its status code. (See 2.5.2 for more details.)

Caveat: because VAUCANSON builds rational expressions on the of words, the Kleen star operator and the weights (introduced in subsection 2.4.3) apply to words and not letters as it is usually the case. For instance 'ab*' is the same rational expression as '(ab)*', but it is different from 'a.b*' or 'a.(b*)'.

2.4.2 Empty word and null series

The default representation of the empty word (identity of the monoïd) is '1' when using characters or pair alphabets. For instance let us try the command expand, that distributes concatenations over disjunctions:

```
$ vcsn-char-b expand -aab '(a+1)(1+b)'
a+ab+b+1
```

Of course if we use '1' as one character in the alphabet, the same symbol cannot be used for representing the empty word. VAUCANSON actually choose the first available representation of the empty word from the following list of candidate symbols: '1', 'e', and '_e', 'eps'.

```
$ vcsn-char-b expand -a01 '(0+e)(e+1)'
0+01+1+e
$ vcsn-char-b expand -a1e '(1+_e)(_e+e)'
1+1e+e+_e
```

For integer alphabets, the empty word is of course always 'e'.

Similarly the symbol used to represent the null series defaults to the first representation from the following list that is compatible with the alphabet: '0', 'z', '_z', 'zero'.

Section 2.5.3 shows how you can actually specify you own representation for these symbols

2.4.3 Weights

Weights are written in braces as in $\{3\}$ '.

For instance the automaton C_1 from subsection 3.2.2 corresponds to the rational expression $(a+b)*.b.({2}a+{2}b)*$.

\$ vcsn-char-z aut-to-exp c1.xml
(a+b)*.b.({2} a+{2} b)*

FIXME: Explain Left/Right weights.

2.4.4 Interactive Definition of Automata

The TAF-KIT command edit-automaton provides a textual interface to define automata interactively. The commands takes the filename of the automata to define or modify in argument. If the files does not yet exist, you should specify the alphabet of your automaton on the command line (using '--alphabet=' or '-a' as will any other command), and the file will be created when you exit the editor. If the file does exist, the alphabet will be read from the file along with the automaton itself, and the file will be overwritten upon exit.

The interface is based on a menu of choices:

```
$ vcsn-char-b edit-automaton --alphabet=ab test.xml
Automaton description:
 States: (none)
  Initial states: (none)
 Final states: (none)
 Transitions: (none)
Please choose your action:
  1. Add states.
  2. Delete a state.
 3. Add a transition.
 4. Delete a transition.
 5. Set a state to be initial.
 6. Set a state not to be initial.
 7. Set a state to be final.
 8. Set a state not to be final.
 9. Display the automaton in Dotty.
  10. Exit.
Your choice [1-10]:
If you enter 1, you will then be prompted for the number of states to add, say 1 again. The state
0 was created. To make it initial select 5, and:
```

```
Your choice [1-10]: 5
For state: 0
```

Likewise to make it final, using choice 7. Finally, let's add a transition:

```
Your choice [1-10]: 3
Add a transition from state: 0
To state: 0
Labeled by the expression: a+b
```

The automaton is generalized, that is to say, rational expressions are valid labels.

On top of the interactive menu, the current definition of the automaton is reported in a textual yet readable form:

```
Automaton description:
States: 0
Initial states: 0
Final states: 0
Transitions:
1: From 0 to 0 labeled by ({1} a)+({1} b)
```

States are numbered from 0, but transitions numbers start at 1. Also, note that weights are reported, although only 1 is valid for Boolean automata.

Finally, hit 10 to save the resulting automaton in the file 'test.xml'.

2.5 Command I/O options

As we said in section 2.3, each TAF-KIT command has to read its input and write its output. In this section we only cover the options that may modify the Input/Output behaviors of commands.

long option	short	purpose	documentation
'alphabet'	'-a'	specify the alphabet of automata or rational ex-	subsection 2.5.1
		pressions	
'alphabet1'	'-a'	specify the first alphabet on transducers	subsection $2.5.1$
'alphabet2'	'-A'	specify the second alphabet on transducers	subsection $2.5.1$
'input'	'-i'	select input format for automata and rational expressions	subsection 2.5.2
'output'	'-o'	select output format for automata and rational expressions	subsection 2.5.2
'parser'	'-p'	fine-tune the symbols used for input and output of rational expressions and automata	subsection 2.5.3

The full list of options can be obtained with vcsn-char-b --help.

2.5.1 Specifying alphabets

When TAF-KIT reads an XML file, there is no need to specify any other information besides the name of the file. For instance when we read 'a1.xml' in section 2.2 and determinized this automaton, we did not have to tell TAF-KIT that the alphabet was $\Sigma = \{a, b\}$. The XML is self-contained and already contains this information.

Here is a situation where specifying an alphabet is mandatory:

\$ vcsn-char-b exp-to-aut aba+a Error: alphabet should be explicitly defined using --alphabet

exp-to-aut is a command that takes a rational expression and converts it into an automaton. To be able to parse the rational expression, VAUCANSON needs to know what alphabet it its using. Here there is no ways it can guess whether the alphabet is $\Sigma = \{a, b\}$ and the '+' is a rational operator or if it is $\Sigma = \{a, b, +\}$ and the '+' is just a letter. Specifying the alphabet can be done using '--alphabet=ab' for instance.

\$ vcsn-char-b exp-to-aut --alphabet=ab aba+a > aut.xml

In practice, the long '--alphabet=' option can be tedious to type and we will often prefer its short equivalent '-a':

\$ vcsn-char-b exp-to-aut -aab aba+a > aut.xml

Character alphabets For characters alphabets (as with the 'char' TAF-KIT instances used in the above examples), the letters of the alphabets can be arbitrary ASCII characters, and need just to be listed after the '--alphabet=' or '-a' option.

When specifying characters alphabets, some characters like '' (space), '"', '(', ')', ''', and '=' have to be escaped with a backslash. For instance the following command will create an automaton that recognize numbers of the form '12,456,789', where a comma must be used as thousand separator: note how the comma must be escaped in the alphabet

```
$ d="(0+1+2+3+4+5+6+7+8+9)"
$ vcsn-char-b exp-to-aut -a'0123456789\,' "($d+$d$d$d+$d$d$d$d)(,$d$d$d$d)*" > numbers.xml
```

Some character alphabets are predefined. These are:

'letters'	The lower case letters $\{a, b, \ldots, z\}$.
'alpha'	The upper and lower case letters $\{a, b, \ldots, z, A, B, \ldots, Z\}$.
'digit'	All digits $\{0, 1, \dots, 9\}$.
'ascii'	All ASCII characters.

This means that '-aletters' is an abbreviation for '-aabcdefghijklmnopqrstuvwxyz'. You get always get the above list by typing 'vcsn-char-b --help'.

Integer alphabets Using integers alphabets, letters must be specified as signed integer (they are represented by the int C++ type), and should be separated by commas. For instance the following commands will construct an automaton that reads any sequence of coins of 1, 2, 5, 10, 20, or 50 cents, as long as the values are increasing.

\$ vcsn-int-b exp-to-aut -a1,2,5,10,20,50 '1*2*5*10*20*50*' > coins.xml

Pair alphabets Pair alphabets should be specified using parenthesis group pairs and commas to separate the constituents of a pair. For instance:

\$ vcsn-char-int-b exp-to-aut -a'(a,1)(b,2)(a,-1)' '(a,-1)(a,1)+(b,2)' > misc.xml

Transducer alphabets Free monoïd products have two alphabets, one for each monoïd. The instances of TAF-KIT that handle transducers consequently support two options '--alphabet1=' and '--alphabet2=', that can be abbreviated respectively '-a' and '-A'.

2.5.2 Input and Output Formats

TAF-KIT can input and output several kind of objects: automata, rational expressions, words, weights and Boolean results.

words are always read as strings given on the command line, and written on standard output.

automata are read from a filename specified on the command line, and output on standard output. VAUCANSON can read automata in two formats: FSMXML (the default), or the textual format of FSM. It can also write automata in these formats, or the 'dot' format, used for graphical output.

- rational expression are by default read as strings given on the command line, and output as strings on standard output. Alternatively rational expression can be read from an FSMXML file whose filename is given on the command line, and output in FSMXML as well.
- weight results (such as the result of the evaluation of a word on an automaton) are simply output as strings on the standard output
- **Boolean results** (such as the result of asking whether an automaton is empty) are returned using the status code of the TAF-KIT instance, so that these commands can be used as conditions in shell scripts.

Changing the format for automata and rational expressions The format used to input automata and rational expressions can be controlled using the '--input=' and '--output=' options (or '-i', '-o' for short). These options control the I/O formats for both automata and rational expressions at once. So for instance using '-ixml' will ask TAF-KIT to read any automaton or rational expression using the FSMXML format. Because rational expression are not supported in as much formats as automata, they will be read or written as text string if an unsupported format is requested.

values for '-i' or '-o'	format for automata	format for rational expressions
(none)	FSMXML	text string
'xml'	FSMXML	FSMXML
ʻfsm'	FSM	text string
'dot' (for output only)	dot	text string

For instance to convert an automaton from XML to dot¹, we would use:

\$ vcsn-char-b identity -odot aut.xml > aut.dot

Verbose Boolean results As said above boolean results are returned using the program's status code using the Unix convention (that is: 0 for *true* and any other value for *false*). The shell makes this value available in the '\$?' variable. The TAF-KIT option '--verbose' or '-v' can be used to request an English interpretation of this value.

```
$ vcsn-int-b is-empty coins.xml
$ echo $?
1
$ vcsn-int-b is-empty -v coins.xml
Input is not empty
```

2.5.3 Specifying writing data

Section 2.4.2 showed how the empty word and null series can have different symbols. VAUCANSON actually allows other symbols used in rational expression to be changed.

Here is the list of named symbols with their meaning and default values:

 $^{^1\}mathrm{dot}$ files can be processed using the GraphViz package

symbol	meaning	default value(s)
'OPAR'	group start	'('
'CPAR'	group end	')'
'PLUS'	disjunction (additive law of the series)	·+'
'TIMES'	concatenation (multiplicative law of the series)	· · ·
'STAR'	Kleene star	·*'
'ONE'	empty word (identity of the monoïd)	`1`,`e`,`_e`,`eps'
'ZERO'	null series	`0`,`z`,`_z`,`zero'
'OWEIGHT'	weight start	$\left\{ \right\}$
'CWEIGHT'	weight end	·},
'SPACE'	space characters (to ignore)	ζ,

The '--parser=' can be used to change the value of the above tokens. Each of them must be defined as a non-empty string. TAF-KIT will check that these tokens do not collide. For instance to use $\{(,)\}$ as alphabet, we should obviously rename the 'OPAR' and 'CPAR' tokens.

The following command creates an automaton that recognizes the words (), (()), (()), (()), (()), (()), (()), (())), etc.

```
$ vcsn-char-b -a'' --parser='OPAR=[ CPAR=]' exp-to-aut '([()]*)' >parens.xml
```

The values of these symbols, which we call the *writing data*, are stored in the XML file, so there is no need to specify this again when working from a file.

```
$ vcsn-char-b aut-to-exp parens.xml
(.(.[).(]*.).)+(.)
$ vcsn-char-b eval parens.xml '(()())'
1
```

Overwriting the writing data When TAF-KIT reads an automaton or a rational expression from an XML file (that contains writing data) or from the internal pipe, it does not need additional information to read its input. However the '--parser=' option can still be used to modify the way the object will be *output*.

Here is an example where we create a rational expression over the alphabet {(,)} using '[' and ']' for grouping, and store it into the file 'p.xml'. We can then convert this file back into a string using either the original writing data that were stored in the file, or overwriting these data with different ones (here using '<' and '>' for grouping).

```
$ vcsn-char-b -a'' --parser='OPAR=[ CPAR=]' identity-exp -oxml '([()]*)' >p.xml
$ vcsn-char-b identity-exp -ixml p.xml
(.[(.)]*.)
$ vcsn-char-b --parser='OPAR=< CPAR=>' identity-exp -ixml p.xml
(.<(.)>*.)
```

the command indentity-exp used in the above example does not apply any algorithm on the rational expression. Its only purpose is to read and write the rational expression using any I/O option supplied on the command-line.²

2.6 An example of \mathbb{Z} -automaton

This part shows some uses of the program vcsn-char-z.

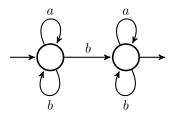


Figure 2.2: \mathcal{B}_1 : When defined over the Boolean semiring $\langle \mathbb{B}, \vee, \wedge \rangle$, \mathcal{B}_1 accepts words with at least one b. When defined over the integer semiring $\langle \mathbb{Z}, +, \times \rangle$, \mathcal{B}_1 counts the number of b in a word.

2.6.1 Counting 'b's

Let's consider \mathcal{B}_1 (Figure 2.2), an N-automaton, *i.e.* an automaton whose label's weights are in N. This time the evaluation of the word w by the automaton \mathcal{B}_1 will produce a number, rather than simply accept or reject w. For instance let's evaluate 'abab' and 'bbab':

```
$ vcsn-char-z eval b1.xml abbb
3
$ vcsn-char-z eval b1.xml abab
2
```

Indeed, \mathcal{B}_1 counts the number of 'b's.

Power

Now let's consider the \mathcal{B}_1^n , where

$$\mathcal{B}_1^n = \prod_{i=1}^n \mathcal{B}_1, n > 0$$

This is implemented by the power function:

\$ vcsn-char-z power b1.xml 4 > b4.xml

The file 'b4.xml' now contains the automaton \mathcal{B}_1^4 . Let's check that the evaluation of the words 'abab' and 'bbab' by \mathcal{B}_1^4 gives the fourth power of their evaluation by \mathcal{B}_1 :

```
$ vcsn-char-z eval b4.xml abbb
81
$ vcsn-char-z eval b4.xml abab
14
```

Quotient

Successive products of an automaton create a lot of new states and transitions.

```
$ vcsn-char-z info b1.xml
States: 2
Transitions: 5
Initial states: 1
Final states: 1
$ vcsn-char-z info b4.xml
States: 16
Transitions: 97
Initial states: 1
Final states: 1
```

²When reading a rational expression, trivial identities are also rewritten. (FIXME: ref).

One way of reducing the size of our automaton is to use the quotient algorithm.

\$ vcsn-char-z quotient b4.xml \/ vcsn-char-z info States: 5
Transitions: 15
Initial states: 1
Final states: 1

Chapter 3

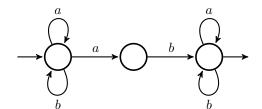
Automaton Repository

VAUCANSON comes with a set of interesting automata that can be used to toy with TAF-KIT (chapter 2) for instance. In this chapter, we present each one of these automata.

3.1 \mathbb{B} automata

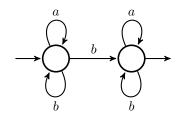
The following automata are predefined files that can be loaded with vcsn-char-b.

3.1.1 'a1.xml' (\mathcal{A}_1)



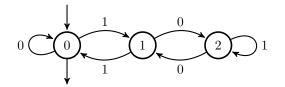
The automaton \mathcal{A}_1 , defined over the alphabet $\Sigma = \{a, b\}$, recognizes any word of Σ^* that contains ab.

3.1.2 'b1.xml'
$$(\mathcal{B}_1)$$



The Boolean automaton \mathcal{B}_1 accepts words with at least one *b*. See also subsection 3.2.1 where \mathcal{B}_1 is defined over $\langle \mathbb{Z}, +, \times \rangle$.

3.1.3 'div3base2.xml'

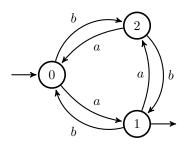


This deterministic automaton is defined on the alphabet $\Sigma = \{0, 1\}$ and recognizes the base-2 representations of all the integers that are divisible by 3.

The source code distribution of VAUCANSON actually comes with a programs called divkbaseb that can generate a divisor by any integer k in any base b. The program is located in 'data/automata/char-b/divkbas in the source tree, and take k and b as arguments.

In the general case the binary digits should be read from left to right. In the case of 'div3base2.xml' the digits can also be read from right to left because the automaton is symmetric.

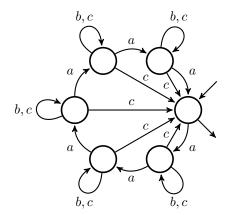
3.1.4 'double-3-1.xml'



This deterministic automaton has three states organized as a double ring. It can read a in one direction and b in the other direction.

The program data/automata/char-b/double_ring can be used to generate a ring of any number of states. It takes that number of states as first argument, and the list of initial states in the remaining arguments. The above example uses 3 states and state 1 as initial state, hence the name 'double-3-1.xml'.

3.1.5 'ladybird-6.xml'

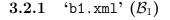


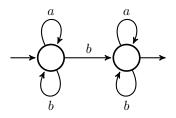
A non-deterministic automaton of 6 states whose determinized version (using the powerset construction) has 2^6 states. We call it ladybird simply because its drawing looks like a *Coccinella*.

The data/automata/char-b/ladybird program can be used to produce ladybirds of any size, just pass the number of states n as first argument. We like to use ladybirds for benchmarking because their determinization always produce the worst case with 2^n states.

3.2 \mathbb{Z} automata

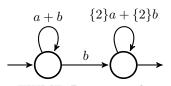
The following automata are predefined files that can be loaded with vcsn-char-z.





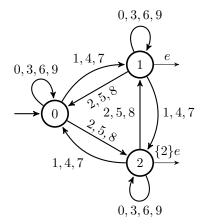
This automaton has the same structure as the one from subsection 3.1.2 but a different type. When defined over the $\langle \mathbb{Z}, +, \times \rangle$ semiring, this automaton is counting the number of b in a word.

3.2.2 'c1.xml'
$$(\mathcal{C}_1)$$



FIXME: Document this automaton.

3.2.3 'div3base10.xml'



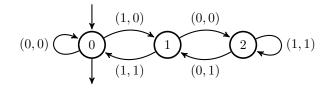
This automaton computes the remainder of an integer division by 3. The integer to divide should be expressed in base 10 and its digits should be read from left to right (although it does not matter in the case of a division by 3).

The program 'data/automata/char-z/divkbaseb', in the source tree, can be used to construct such a divisor by k in any base b.

3.3 **B** FMP

The following transducers are predefined files that can be loaded with vcsn-char-fmp-b.

3.3.1 'quot3base2.xml'



A transducer computing the quotient by 3 of a binary number, read from left to right. The program 'data/automata/char-fmp-b/quotkbaseb', in the source tree, can be used to construct such a divisor by k in any base b.

3.3.2 't1.xml'

